ASN Sr. Sec. School

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Class 12

HOLIDAY HOMEWORK

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MATHS PART2

CHAPTER 6

APPLICATIONS OF DERIVATIVES

POINTS TO REMEMBER

- **Rate of Change :** If x and y are connected by $y = f(x)$ then $\frac{dy}{dx}$ represents the rate of change of y *w.r.t.* x.

- Equation of tangent to the curve $y = f(x)$ at the point $P(x_1, y_1)$ is given by $y - y_1 = \left.\frac{dy}{dx}\right|_P (x - x_1)$.

  Similarly equation of normal is $y - y_1 = -\frac{1}{\left.\frac{dy}{dx}\right|_P}(x - x_1)$.

  The angle of intersection between two curves is the angle between the tangents to the curves at the point of intersection. $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$, where m_1, m_2 are slopes of tangent at the point of intersection P.

- A function $f(x)$ is said to be strictly monotonic in (a, b) if it is either increasing or decreasing in (a, b).

- A function $f(x)$ is said to be strictly increasing in (a, b) if $\forall\ x, x_2$ in (a, b) s.t. $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$. Alternatively, $f(x)$ is increasing in (a, b) if $f'(x) > 0\ \forall\ x \in (a, b)$.

- A function $f(x)$ is said to be strictly decreasing in (a, b) if $\forall\ x_1, x_2$ in (a, b) s.t. $x_1 < x_2$ Þ $f(x_1) > f(x_2)$. Alternatively, $f(x)$ is strictly decreasing in (a, b) if $f'(x) < 0\ \forall\ x \in (a, b)$.

- A function $f(x)$ is said to have local maximum value at $x = c$, if there exists a neighbourhood $(c - \delta)$, $(c + \delta)$ of c, s.t. $f(x) < f(c)\ \forall\ x \in (c - \delta, c + \delta)\ x \neq c$. Similarly, local minimum value can be defined.

- Local maximum and local minimum values of $f(x)$ may not be maximum and minimum value of $f(x)$.

- **Critical Point :** A point c is called critical point of $y = f(x)$ if either $f'(c) = 0$ or $f'(c)$ does not exist.

Some useful results :

Figure Surface area	Curved S.A.	Total S.A.	Volume
Sphere	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Cone	πrl	$\pi rl + \pi r^2$	$\frac{1}{3}\pi r^2 h$
Cylinder	$2\pi rh$	$2\pi rh + 2\pi r^2$	$\pi r^2 h$

VERY SHORT ANSWER TYPE QUESTIONS (1 Mark Each)

1. Write minimum value of $f(x) = x^2 + x + 1$ in $[0, 1]$.

*2. If Rolle's theorem in applicable for the function $f(x) = x^2 - 3x + 1$ in $[-1, 4]$ then find the real no 'c' verifying Rolle's theorem.

3. Find the interval where $f(x) = \cos x$ defined in $[0, 2\pi]$ is decreasing.

4. Find the interval where $f(x) = x^2, x \in (-\infty, \infty)$ is decreasing.

*5. For what value (s) of λ, the function, $f(x) = \sin x - 3\lambda x$ is always strictly increasing.

*6. Write the interval in which $f(x) = x^x$ is increasing (where $x > 0$).

7. Examine if $f(x) = x^9 + 2x^5 + 3x^3 + 1$ is increasing or decreasing $(0, \infty)$.

*8. Write the least value of $f(x) = x + \frac{1}{x}, (x > 0)$.

9. Write the maximum value of $f(x) = \frac{1}{x^2 - 2x + 3}$ in $[0, 2]$.

*10. Find the maximum and minimum value of $f(x) = |2 \sin 2x + 3|$.

*11. On the curve $f(x) = \frac{3}{2}x^2$, find the points at which tangent is parallel to the chord joining the points $A\left(-1, \frac{3}{2}\right)$ and B(2, 6).

*12. If the tangent to the curve at a point P is perpendicular to x-axis, then what is the value of $\frac{dy}{dx}$ (if it exists) at the point P.

*13. If normal to the curve at a point P on $y = f(x)$ is parallel to y-axis, then write the value of $\frac{dy}{dx}$ at P.

14. What is the slope of the tangent to the curve $y = x^2$ at $(-1, 1)$.