ASN Sr. Sec. School

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Class 12

HOLIDAY HOMEWORK

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MATHS PART1

CHAPTER 5

DIFFERENTIATION

POINTS TO REMEMBER

- **Continuity of a Function :** A function $f(x)$ is said to be continuous at $x = c$ if $\lim_{x \to c} f(x) = f(c)$ *i.e.*, L.H.L. at $x = c$ = R.H.L. at $x = c = f(c)$.

  $f(x)$ is continuous in $[a, b]$ iff :

  $f(x)$ is continuous at $x = c \ \forall \ c \in [a, b]$.

- If f and g are two continuous function then $f + g$, $f - g$, $f.g.$, cf, $|f|$ are all continuous function.

- $\frac{f}{g}$ is continuous at $x = a$ provided $g(a) \neq 0$.

- Every polynomial function is a continuous function.

- $f(x)$ is said to be derivable at $x = c$ iff $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ exists and value of this limit is denoted by $f(c)$.

$$\frac{d}{dx}(u.v) = u.\frac{dv}{dx} + v.\frac{du}{dx}, \quad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}.$$

- If y is a function of u and 'u' is function of x them, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ [chain rule].

- If $x = \phi_1(t)$, $y = \phi_2(t)$ then $\frac{dy}{dx} = \frac{\phi_2'(t)}{\phi_2'(t)} = g(t)$ say then $\frac{d^2y}{dx^2} = g'(t).\left(\frac{dt}{dx}\right)$.

- **Rolle's theorem :** If $f(x)$ is continuous in $[a, b]$ and derivable in (a, b) and $f(a) = f(b)$ then there exists atleast one real no $c \in (a, b)$ $s.t. f'(c) = 0$.

- **L.M.V.T. :** If $f(x)$ is continuous in $[a, b]$ and derivable in (a, b) then $\exists$ atleast one point $c \in (a, b)$ $s.t.$ $f'(c) = \frac{f(b) - f(a)}{b - a}$.

VERY SHORT ANSWER TYPE QUESTIONS (1 Mark Each)

1. At what point $f(x) = |3x - 5|$ is not differentiable.

*2. What is derivative of $f(x)$ if $f(x) = |x|$, $x \neq 0$.

3. At what point $f(x) = |\,|x - 1| - 2|$ is not differentiable.

4. Write the points of discontinuity of $f(x) = \dfrac{x^2 + x + 1}{x^2 - 5x + 6}$.

5. Write all the points of discoutinuity of $f(x) = [x]$, where $[x]$ is the greatest integer function.

*6. At what point, $f(n)$ is discontinuous where, $f(x)$ is signum function defined as

$$f(x) = \begin{cases} \dfrac{x}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}.$$

7. Write the interval in which $f(x)$ is continuous where $f(x) = e^x \log x$.

8. For what value of λ, $f(x) = \begin{cases} \lambda\, x^2 + 1 & x \geq 2 \\ -1 & x < 2 \end{cases}$ is continuous on R.

*9. Write the interval in which $f(x)$ is continuous where $f(x) = \dfrac{\log x}{\sqrt{1 - 9x^2}}$.

*10. Write the value of K given that $f(x) = \begin{cases} \dfrac{\sin x}{x} & x \neq 0 \\ K & x = 0 \end{cases}$.

*11. What is the derivative of x^6 with respect to x^3.

*12. What is the Derivative of $f(\log x)$ is $f(x) = \log x$.

13. If Mean value theorem holds for $f(x) = e^x$, $x \in [0, 1]$, then for what value of x, $M\ V\ T$ is verified.

*14. What is $\dfrac{d}{dx}\left(\sin^{-1} x + \cos^{-1} x\right)$ if $-1 \leq x \leq 1$.

15. Given $g(0) = 2$ and $f(x) = x\, g(x)\, f'(x)$ and $g'(x)$ exist then what is $f'(0)$.

16. Write the derivative of the following function *w.r.t.* x.

(a) $\tan^{-1}\left(\dfrac{\sqrt{a} - \sqrt{x}}{1 + \sqrt{a}\,\sqrt{x}}\right)$, $x > 0$, $a > 0$

(b) $\sec^{-1}\left(\dfrac{1}{2x^2 - 1}\right)$.

(c) $\log_5 (3x - 5)$.

*(d) $\tan^{-1}\left(\dfrac{x}{1 + 12x^2}\right)$.

(e) $\tan^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right)$.

(f) $e^{\tan^{-1} x^2}$.

(g) $\cos x^3 \cdot \sin^2 x^5$.

(h) $\sqrt{e^{\sqrt{x}}}$.

(i) $\sin^{-1}(x\sqrt{x})$ $0 \le x \le 1$. (j) $\sin^{-1}(\sqrt{\cos x})$.

SHORT ANSWER TYPE QUESTION (4 Marks)

Discuss the continuity of the following functions at indicated points.

17. $f(x) = \begin{cases} \frac{x}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}$ at $x = 0$.

18. $f(x) = \begin{cases} \frac{x - |x|}{x} & x \neq 0 \\ 2 & x = 0 \end{cases}$ at $x = 0$.

19. $f(x) = \begin{cases} \frac{e^x - 1}{\log(1 + 3x)} & x \neq 0 \\ 5 & x = 0 \end{cases}$ at $x = 0$.

20. $f(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ at $x = 0$.

21. $f(x) = |x - 1| + |x + 1|$ at $x = -1$, $x = 1$.

22. $f(x) = \begin{cases} \frac{\sin x}{x} & x < 0 \\ x + 1 & x \ge 0 \end{cases}$ at $x = 0$.

23. $f(x) = \begin{cases} \frac{\sqrt{1+x} - \sqrt{1-x}}{\delta m\, x} & x \neq 0 \\ 1 & x = 0 \end{cases}$ at $x = 0$.

24. $f(x) = \begin{cases} \frac{|\sin x|}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$ at $x = 0$.

25. For what value of K, $f(x) = \begin{cases} 3 - 2x & 0 < x < 2 \\ 4x^2 - 3kx & 2 \le x < 5 \end{cases}$ is continuous in it's domain.

26. For what values of a and b

$$f(x) = \begin{cases} \frac{x+2}{|x+2|} + a & \text{if } x < -2 \\ a + b & \text{if } x = -2 \\ \frac{x+2}{|x+2|} + b & \text{if } x > -2 \end{cases}$$

is continuous at $x = 2$.