ASN Sr. Sec. School

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Class 12

HOLIDAY HOMEWORK

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MATHS PART3

*27. If $f(x) = \dfrac{1}{1-x}$ then find the point of discontinuity if any of $f[f\{f(x)\}]$.

28. Prove that $f(x) = |x - 2|$ is continuous at $x = 2$ but not differentiable at $x = 2$.

29. For what value of K, $f(x) = \begin{cases} \dfrac{3x - \tan x}{5x - \sin x} & x < 0 \\ K & x = 0 \\ 3x^2 - 4x + \dfrac{1}{2} & x < 0 \end{cases}$ is continuous at $x = 0$.

30. Show that $f(x) = x - [x]$ is discontinuous at $x = 2$. Also discuss the countinuity at $x = \dfrac{5}{2}$, where [ ] represents greatest integer function.

31. Check the differentiability of $f(x) = |x - 1| + |x - 2|$ at $x = 2$.

*32. If $f(x) = \begin{cases} x^p \sin \dfrac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ is differentiable at $x = 0$, then find value of p.

33. For what value of a and b $f(x) = \begin{cases} 5 & \text{if } n \le 2 \\ ax + b & \text{if } 2 < x < 10 \\ 21 & \text{if } x \ge 0 \end{cases}$ is continuous.

34. If $y = (\log x)^x + x^{\log x}$ then find $\dfrac{dy}{dx}$.

35. If $y = \dfrac{1}{2}\left[\tan^{-1} \dfrac{2x}{1-x^2} + 2\tan^{-1}\dfrac{1}{x}\right]$ find $\dfrac{dy}{dx}$.

36. If $y = \sin\left[2\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\right]$ then find $\dfrac{dy}{dx}$.

37. If $x^{2/3} + y^{2/3} = a^{2/3}$ then show that $\dfrac{dy}{dx} = -\left(\dfrac{y}{x}\right)^{\frac{1}{3}}$.

38. If $y = \tan^{-1} x$, show that $(1 + x^2)\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} = 0$.

39. If $f(x) = \log(x^x + \sec^3 x)$, find $f(x)$.

40. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ then prove that $\dfrac{dy}{dx} = \dfrac{-1}{(1+x)^2}$, $x \neq y$.

*15. If the tangent to the curve $y = 2x^2 - x$ at any point P is parallel to the line $x - y = 0$, then find the coordinates of P.

16. If the tangent to the curve $x = at^2$, $y = 2at$ is perpendicular to x-axis then write the coordinates of the point of contact of tangent.

*17. If curves $y = 3e^{2x}$ and $y = be^{-2x}$ cut each other orthogonally, then find b.

*18. At which point on $y^2 = 4x$, the tangent makes an angle of 45° with the positive direction of .

*19. If $kx + y = P$ is normal to the curve $y^2 = 12x$ at (3, 6) then what is value of k.

20. How many extreme values [maximum or minimum] are there of $f(x) = x$.

21. What is equation of normal to the curve $y = \sin x$ at origin.

SHORT ANSWER TYPE QUESTIONS (4 Marks Each)

22. Sand is pouring out from a pipe at the rate of 12 Cu cm/s. The falling sand forms a cone one the ground in such a way that the height of the cone is always one sixth of the radius of the base. How fast is the height of sand cone increasing when the height is 4 cm.

23. A particle moves along the curve $y = x^5 + 2$. Find the points on the curve at which y co-ordinate is changing 5 times as fast as the x co-ordinate.

24. Find points of local maxima/minima for $f(x)$. If $f(x) = \sin x - \cos x$ where $0 < x < 2\pi$. Also find the local maximum or minimum values.

25. Find the intervals in which the function $f(x) = x^4 - \frac{x}{3}$ is increasing or decreasing.

*26. If $f(x) = x^2 - 2x + 3$ then using differentials, find the approximate value of $f(1.9)$.

27. Find the value (s) of a for which :

   (i) $f(x) = x^3 - ax$ is increasing on R.

   (ii) $g(x) = \sin x + ax$ is increasing on R.

28. If radius of right circular cone is increasing at the rate of 10π cm^3/sec, find the rate at which the height of the cone is hanging at the instant when radius 5 cm and height 4 cm.

*29. Find the least value of the function. $f(x) = ax + \frac{b}{x}$, $(a, b\ x > 0)$.

30. For the curve $y = 2x^3 - 3x^2$, find all the points on the curve at which the tangent passes through the origin.

31. Prove that the function :

$$f(x) = x^{50} + \sin x - 1 \text{ is strictly increasing on } \left(\frac{\pi}{2}, \pi\right).$$

LONG ANSWER TYPE QUESTION (6 Marks Each)

51. Show that the point (1, 3) on $y = x^2 + 2$ is nearest to the point (3, 2).

52. A point on the hypotenuse of a right triangle is at a distance 'a' and 'b' from the sides of the triangle. Show that the minimum length of the hypotenuse is $(a^{2/3} + b^{2/3})^{3/2}$.

53. If the length of three sides of a trapezium other than base are equal to 10cm, then find the area of trapezium when it is maximum.

54. A given quantity of metal is to be cast into half cylinder with a rectangular base and semi-circular ends. Show that when total surface areas is minimum, the ratio of length of cylinder to the diameter of its semi-circular ends is $\pi : (\pi + 2)$.

55. Show that $f(x) = \sin^4 x + \cos^4 x$, $x \in [0, \pi/2]$ is increasing on $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ and decreasing on $\left[0, \frac{\pi}{4}\right]$.

56. Find the interval in which $f(x) = 2 \log (x - 2) - x^2 + 4x + 1$ is increasing or decreasing,

57. Find the equation of tangent to the curve $y = x^3 - 1) (x - 2)$ at the points where the curve cuts the x–axis.

58. Show that the semi-verticle angle of a cone of maximum volume and given height is $\tan^{-1}\sqrt{2}$.

59. Prove that the radius of the right circular cylinder of greatest curved surface which can be inscribed in a given cone is half of that of the cone.

60. A rectangular sheet of tin 45 cm × 24 cm is to be made into a box without top by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum?

61. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the lengths of the two pieces so that the combined area of the square and the circle is minimum?

62. For a given curved surface of a right circular cone when volume is maximum, prove that semi-vertical angle is $\sin^{-1}\left(\frac{1}{3}\right)$.

63. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle α is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

64. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

65. A jet of an enemy is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3, 2). What is the nearest distance between the soldier and the jet?